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Generalized thermoelastic interactions in a hollow cylinder with temperature-dependent material properties

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Abstract

In the present work, the generalized thermoelastic interactions in a hollow cylinder with one relaxation time are considered. The modulus of elasticity are taking as function of temperature. Due to the nonlinearity of the governing equations, finite element method is adopted to solve such problem. The exact solution in the case of temperature-independent is discussed explicitly. Numerical results for the temperature distribution, displacement and radial and hoop stresses represented graphically. The accuracy of the finite element method validated by comparing between the finite element and exact solutions for temperature-independent.

Key words : Nonlinear thermoelasticity, Lord-Shulman's theory, Finite element method, Hollow cylinder, Temperature-dependent, Exact solution

1. Introduction

In fact, the change of body temperature has an effect on the strain/stress fields and conversely, i.e. mechanical action, and corresponding strain produce a temperature field. The numerical value of thermal conductivity varies with temperature, especially if a region of change of temperature is large. So, the thermal conductivity, and may be the heat capacity, should be considered temperature-dependent in most of the practical engineering problems. Serious attention paid to the generalized thermoelasticity theories in solving thermoelastic problems in place of the classical uncoupled/coupled theory of thermoelasticity. The theory of couple thermoelasticity was extended by Lord and Shulman (LS) (Lord and Shulman 1967) and (Green and Lindsay 1972) by including the thermal relaxation time in constitutive relations. The anisotropic case was later developed by (Dhaliwal and Sherief 1980).

Based on these generalized theories, a large number of efforts have been devoted to investigating generalized dynamic problems. Several experimental studies by Kaminski (Kaminski 1990) and (Tzou 1995). Subsequently, several investigations (Abbas 2014e, Abbas 2014f, Abbas 2014b, Abbas 2014a, Abbas 2014d, Abbas 2014c, Abbas 2014g, Abbas and Kunnar 2014, Abbas and Zenkour 2014b, Abbas and Zenkour 2014a, Zenkour and Abbas 2014c, Zenkour and Abbas 2014b, Zenkour and Abbas 2014a, Abbas 2015, Abbas and Zenkour 2015) are carried out based on different generalized theories of thermoelasticity. In most of the problems, the material properties of the medium are taken to be constant. Modern structure elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements. (Suhara 1918) solved the thermoelastic problems of hollow

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circular cylinder of which only the shear modulus was temperature-dependent. Since his study, many investigators studied temperature-dependent. (Ezzat, El-Karamany et al. 2004) investigated problem in generalized thermoelasticity with the modulus of elasticity dependent with temperature. (Youssef 2005a, Youssef 2005b, Youssef and Al-Harby 2007) has many contributions for temperature-dependent properties of materials.

The aim of the present paper is to investigate a problem of a hollow cylinder, whose material properties like modulus of elasticity and thermal conductivity vary with temperature, in the context of generalized thermoelasticity theory with one relaxation parameter. An application of a hollow cylinder is investigated where the inner surface is traction free and subjected to a decaying-with-time thermal field, while the outer surface is traction free and thermally isolated. Results are displayed graphically and compared with the results obtained for temperature-independent material properties.

2. Governing equations

In the context of the LS-theory, we consider an isotropic elastic medium with temperature dependent material properties. The basic equations and constitutive relations can be presented in a unified form as (Lord and Shulman 1967)

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj} - \gamma T_{,i}. \tag{1}$$

The equation of heat conduction

$$\left(k T_{,i}\right)_{,i} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho c_e T + \gamma T_0 u_{j,j}\right). \tag{2}$$

The constitutive equations are given by

$$\tau_{ij} = \lambda u_{i,j}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) - \gamma\left(T - T_0\right)\delta_{ij}. \tag{3}$$

where ρ is the mass density; T the temperature change of a material particle; T_0 the reference uniform temperature of the body; u_i the displacement vector components; e_{ij} the strain tensor; τ_{ij} the stress tensor; c_e the specific heat at constant strain; γ the thermal elastic coupling tensor in which $\gamma = (3\lambda + 2\mu)\alpha_t.$; k the thermal conductivity; λ , μ are elastic parameters; τ_o is a relaxation time. For temperature dependent material, we will suppose that

$$\lambda = \lambda_o f(T),\ \mu = \mu_o f(T),\ k = k_o f(T),\ \rho = \rho_o f(T),\ \gamma = \gamma_o f(T),$$

where $\lambda_o,\ \mu_o, k_o,\ \rho_o,$ and γ_o are considered constants, $f(T)$ is a continuous and non-dimensional function in the domain $0 \le T - T_0 < \infty.$ In case of temperature-independent material properties $f(T) = 1$ and $\lambda = \lambda_o,\ \mu = \mu_o, k = k_o,\ \rho = \rho_o,$ and $\gamma = \gamma_o$. Therefore from equations (1)-(3) we have the system of nonlinear partial differential equations:

$$\rho_o \frac{\partial^2 u_i}{\partial t^2} f(T) = \left[(\lambda_o + \mu_o)u_{j,ji} + \mu_o u_{i,jj} - \gamma_o T_{,i}\right]_{,j} f(T) + \left(f(T)\right)_{,j}\left[(\lambda_o + \mu_o)u_{j,ji} + \mu_o u_{i,jj} - \gamma_o T_{,i}\right], \tag{4}$$

$$\left(k_o f(T) T_{,i}\right)_{,i} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho_o c_e T f(T) + \gamma_o T_0 f(T) u_{j,j}\right). \tag{5}$$

$$\tau_{ij} = \left[\lambda_o u_{i,j}\delta_{ij} + \mu_o\left(u_{i,j} + u_{j,i}\right) - \gamma_o\left(T - T_0\right)\delta_{ij}\right] f(T). \tag{6}$$

3. Problem formulation

We consider an isotropic hollow cylinder with internal radius a and external radius b. The inner surface is traction free and subjected to a decaying-with-time thermal field, while the outer surface also is traction free but thermally isolated. We introduce the cylindrical polar coordinates (r,θ,z) with the z-axis lying along the axis of the cylinder. Due to symmetry, the functions considered depending on the radial distance r and the time t where $a \le r \le b$. The displacement vector has the components

$$u_r = u(r,t), \qquad u_\theta(r,t) = 0, \qquad u_z(r,t) = 0. \tag{7}$$

Rishin et al. (Rishin, Lyashenko et al. 1973) investigated the relationship between modulus of elasticity of several sprayed coatings and temperature, and they found the modulus of elasticity decreases monotonically with the increasing of temperature. For simplicity and without loss of generality, we assume:

$$f(T) = 1-\alpha\left(T-T_0\right), \tag{8}$$

where α is an empirical material constant. Then, the equations (4-8) yield the following non-linear equations:

$$\frac{\partial \tau_{rr}}{\partial r}+\frac{1}{r}\left(\tau_{rr}-\tau_{\theta\theta}\right)=\rho_o\left(1-\alpha\left(T-T_0\right)\right)\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\frac{k_o}{r}\frac{\partial}{\partial r}\left(r\left(1-\alpha\left(T-T_0\right)\right)\frac{\partial T}{\partial r}\right)= \left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left[\left(1-\alpha\left(T-T_0\right)\right)\left(\rho_o c_e T+\gamma_o T_0\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right)\right], \tag{10}$$

with

$$\tau_{rr}=\left(1-\alpha(T-T_0)\right)\left[\left(\lambda_o+2\mu_o\right)\frac{\partial u}{\partial r}+\lambda_o\frac{u}{r}-\gamma_o(T-T_0)\right], \tag{11}$$

$$\tau_{\theta\theta}=\left(1-\alpha(T-T_0)\right)\left[\lambda_o\frac{\partial u}{\partial r}+\left(\lambda_o+2\mu_o\right)\frac{u}{r}-\gamma_o(T-T_0)\right]. \tag{12}$$

For our convenience, the following non-dimensional variables and notations are used:

$$(r',u')=c_1\chi(r,u),\ \left(t',\tau_o'\right)=c_1^2\chi(t,\tau_o),\ \left(\tau'_{rr},\tau'_{\theta\theta}\right)=\frac{1}{\lambda_o+2\mu_o}\left(\tau_{rr},\tau_{\theta\theta}\right), \\ T'=\frac{T-T_0}{T_0},\ c_1=\sqrt{\frac{\lambda_o+2\mu_o}{\rho}},\ \chi=\frac{\rho_o c_e}{k_o}. \tag{13}$$

In terms of the non-dimensional quantities defined in equation (13), the above governing equations reduce to (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}-\xi_2\frac{\partial T}{\partial r}-\frac{\beta}{1-\beta T}\frac{\partial T}{\partial r}\left(\frac{\partial u}{\partial r}+\xi_1\frac{u}{r}-\xi_2 T\right)=\frac{\partial^2 u}{\partial t^2}, \tag{14}$$